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RECENT RESEARCH IN COMPOSITE AND SANDWICH PLATE DYNAMICS.(U)
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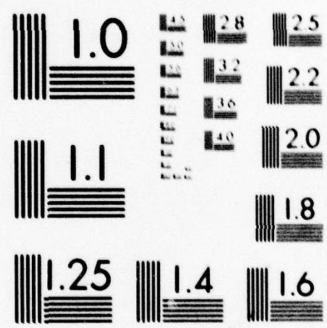

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RECENT RESEARCH IN COMPOSITE AND SANDWICH PLATE DYNAMICS

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Abstract - This paper surveys literature concerning dynamics of plate-type structural elements of either composite material or sandwich construction. Papers from 1976 through early 1979 are reviewed. Special attention is given to rectangularly orthotropic, cylindrically orthotropic, and anisotropic plates; laminated plates; thick and sandwich plates; and nonlinearities. Free vibration, harmonic and random forced vibration, thermally and flow induced vibration (flutter), and impact are also treated.

The fundamentals of the mechanics of composite and laminated plates have been discussed in a previous survey [1] and are not repeated. No books have yet been published in the subject area; the survey thus consists largely of papers in the open literature and some reports. The following topics are excluded: strictly in-plane motion, acoustic wave transmission, and failure due to impact and fatigue loadings.

SMALL-DEFLECTION MOTION OF THIN, SYMMETRICALLY LAMINATED, RECTANGULARLY ORTHOTROPIC PANELS

These panels are the simplest and most extensively investigated of thin composite-material plates and are often called orthotropic plates. The panels can be an aligned single layer, an aligned parallel-ply laminate, or a symmetrical cross-ply laminate. Laminate symmetry about the plate midplane ensures the absence of bending-stretching coupling; aligned orthotropic material symmetry prevents shear-normal coupling. Thus, in the absence of applied shear and in-plane loading, the governing equation of motion can be written as

$$D_{11}w_{,xxxx} + 2(D_{12} + 2D_{66})w_{,xxyy} + D_{22}w_{,yyyy} + \rho h w_{,tt} - p = 0 \qquad (1)$$

The D_{ij} are the plate flexural and twisting rigidities, h is the plate thickness, p is the normal pressure, t is time, w is the plate deflection, $(\ )_{,xxyy}$ denotes $\partial^4(\ )/\partial x^2 \partial y^2$, x and y are rectangular position coordinates coinciding with the material-symmetry directions, and ρ is the mean density.

In terms of energy the analogous expression is

$$U_i + U_e = T \qquad (2)$$

The internal strain energy U_i, work U_e done by the external force, and kinetic energy T are given respectively as

$$\begin{aligned} U_i &= (1/2) \iint (D_{11}w^2_{,xx} + 2D_{12}w_{,xx}w_{,yy} + D_{22}w^2_{,yy} + 4D_{66}w^2_{,xy})\,dx\,dy \\ U_e &= -\iint p\,w\,dx\,dy \\ T &= (1/2) \iint \rho\,h\,w^2_{,t}\,dx\,dy \end{aligned} \qquad (3)$$

Free Vibration

A method for deducing orthotropic solutions from corresponding isotropic solutions was introduced independently in Japan [2, 3] and in the U.S. [1, 4]. This approach enables the vast literature on free vibration of isotropic plates of various planforms to be utilized to predict approximate natural frequencies for orthotropic plates of the same planforms. The accuracy of the method has been verified for the specific geometries listed in the Table. The effects of biaxial inplane loads (tensile or compressive) were later incorporated into the method [5], thus generalizing previous work on buckling of orthotropic plates [6, 7].

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Table. Applications of Isotropic-to-Orthotropic Deduction Technique

Planform Geometry	Boundary Conditions	Reference Number
Rectangular	Fifteen Combinations of clamped and simply supported	1, 4
Parallelogrammic	Simply supported	2
Elliptic	Clamped	1, 3, 4
Right triangular	Clamped and simply supported	1, 4

Insofar as the development of relatively new methods of analysis is concerned, Dharmarajan and Chou [8] extended the method of constant deflection lines – originated for deflection [9] and free vibration of isotropic plates [10] – to orthotropic plates. They applied it to both clamped-edge and simply-supported-edge elliptic-planform plates. The main disadvantage of this method is that it can be used to determine only the fundamental frequency.

Another method, not actually new but seldom applied to vibration of plates, is the Kantorovich method [11], known as the Lévy method in static plate deflection theory [12]. This method has recently been applied to free vibration of orthotropic plates [13]. Solutions were obtained by finite differences with respect to one position coordinate. Uniform-thickness rectangular plates with seven different boundary conditions and linearly tapered rectangular plates with simply supported edges were studied. Values for the first six natural frequencies were in fairly close agreement with those obtained using other methods.

Vijayakumar and Ramaiah [14] used a variation [16] of Bolotin's asymptotic method [15] to obtain an initial solution; this solution became the trial function in the Rayleigh and Rayleigh-Ritz methods. The results for clamped rectangular plates were compared with those obtained previously. Compared with 36-term Rayleigh-Ritz results [17], the modified Bolotin estimates for the fundamental frequency were as much as 4.8% in error (always on the low side); the results with the variation [14] are only 0.32% in error (sometimes lower, other times higher). It is significant that a well-known simple formula [18] gave frequency values as much as 50% too high.

In addition to work mentioned above [13], varying thickness plates have also been investigated; approximate fundamental-frequency expressions were derived for simply supported [19] and clamped [20] edge conditions.

Sakata [21] treated a multi-bay continuous rectangular panel. He considered simple supports at the y=0 and y=b edges, free or elastically restrained edges at x=0 and x=a, and simple supports at the intermediate supports x=a/N, 2a/N, .., (N-1)a/N.

Laura and Luisoni [22] used the Rayleigh-Ritz method in conjunction with polynomial modal functions to analyze a rectangular plate with different values of elastic restraint at the edges and subjected to in-plane forces. Beam functions were used in conjunction with the Rayleigh-Ritz method to treat in-plane-loaded rectangular plates with various combinations of boundary conditions [23]. A conformal-mapping technique used to analyze irregularly shaped plates under in-plane forces [24] led to numerical results for the case of a rectangularly orthotropic plate of circular planform. However, the results appear to contain some numerical error [25].

With the growing industrial importance of the finite-element method, it is not surprising that its use in composite-plate vibration analyses is increasing. A simplified mixed rectangular element has been applied to free vibration of rectangular plates [26]. A combination of triangular and rectangular elements have been used for the vibration of a cantilever plate representative of a delta-planform airplane wing [27].

Forced Vibration

Relatively few investigations on forced vibration have been reported recently, except for some analytical work [28] and experimental work [29] on respnse to random excitation.

An equivalent viscous damping approach was used to predict response of a simply-supported rectangular plate to a sinusoidal point load normal to the plate at its center [30].

Two investigations concerned with thermally induced vibrations were on rectangular [31] and equilateral triangular [32] plates. The latter is severely limited by the assumption that $2(D_{12} + 2D_{66}) = (D_{11} D_{22})^{1/2}$. Related work on the effect of thermal stresses induced by steady-state sinusoidal loading has been reported [33].

SMALL-DEFLECTION MOTION OF THIN, SYMMETRICALLY LAMINATED, RECTANGULARLY ANISOTROPIC PANELS

Relatively very little work has been conducted recently. The governing equation of motion for this class of panel is identical to that in equation (1) with the addition of two bending-twisting coupling terms on the left side: $4D_{16}w_{,xxxy}$ and $4D_{26}w_{,xyyy}$.

In optimization studies of rectangular plates having the symmetric balanced angle-ply lamination arrangement, for instance $\theta/-\theta/-\theta/\theta$, both simply supported [34] and clamped [35] edges were considered. In each case the optimal lamination angle (θ) depends upon the plate aspect ratio as well as the laminate material.

Laura and Grossi [36] considered rectangular plates with supported edges having elastic rotational constraints. However, the latter conditions are satisfied only approximately. Solution was obtained by the Rayleigh-Ritz method with polynomial modal functions. For cases in which comparisons with previous solutions were possible, sufficiently good agreement for engineering purposes was obtained.

In many aerospace, marine, and automotive applications, cutouts are necessary for lightening and access. Thus, work on rectangular panels with rectangular cutouts is of practical importance [37, 38]. The cutout is assumed equivalent to a certain displacement-dependent external loading. Both simply-supported [37] and clamped [38] edges and sinusoidally forced and free vibrations were treated. Results in graphical form depicted the effects of plate material, orientation, and plate and cutout geometry.

In analyses of the flutter of rectangular panels under high-Mach-number supersonic conditions, numerical results were presented for boron-epoxy, graphite-epoxy, and boron-aluminum unidirectional materials at various orientations [39].

SMALL-DEFLECTION MOTION OF THIN, UNSYMMETRICALLY LAMINATED, RECTANGULARLY ORTHOTROPIC AND ANISOTROPIC PANELS

Finite-element analyses are now being used for this class of plate [40, 41]. The application of the NASTRAN CQDPLT element to rectangular-planform cantilever plates gave excellent agreement with experimental results for boron-epoxy panels [40]. For this set of boundary conditions, the reduced stiffness simplification was shown to be adequate. In this approach the direct bending-stretching coupling terms are omitted, but the actual bending stiffness matrix $[D_{ij}]$ is replaced by the reduced one $[D_{ij}^*]$ defined as follows

$$[D_{ij}^*] = [D_{ij}] - [B_{ij}]\,[A_{ij}]^{-1}\,[B_{ij}]$$

NASTRAN CTRIA 2 was used to analyze the free vibration of cantilever compressor blades [41]. Results were in good agreement with resonant frequencies and nodal patterns obtained experimentally by holography.

The Rayleigh-Ritz method was used to determine the resonant frequencies of clamped cross-ply laminated plates of circular planform [42].

The Galerkin method was used to obtain Fourier-form solution for the flutter of arbitrarily laminated thin plates [43]. Graphical results showed the effects of lamination angle, lamination arrangement, and flow direction on the flutter parameter for plates laminated of glass-epoxy, boron-epoxy, and graphite-epoxy.

SMALL-DEFLECTION MOTION OF THIN, SYMMETRICALLY OR UNSYMMETRICALLY LAMINATED, CYLINDRICALLY ORTHOTROPIC OR ANISOTROPIC PANELS

Apparently it would be difficult to manufacture a uniform-thickness plate that is cylindrically orthotropic; i.e., having the material-symmetry axes oriented in the radial and circumferential directions. Nevertheless, vibrational analyses of such plates, either single-layer/symmetrically laminated [44-46] or arbitrarily laminated [47] have been reported. It would seem to be even more difficult to construct a cylindrically anisotropic plate; i.e., having a full array of elastic properties (no symmetries or zero values) with respect to a cylindrical coordinate system. However, the vibrations of this class of plate have also been analyzed [49].

Prathap and Varadan [44] considered the axisymmetric free vibration of a solid circular plate of cylindrically orthotropic material. Comparison of the results obtained by the Rayleigh energy method (called the Lagrangian method by the authors) with those obtained by the Galerkin method revealed the source of some discrepancies in some existing analyses of this problem. The method of Frobenius for the nonaxisymmetric case [45] was used to obtain results that have been criticized [44].

Ramaiah and Vijakumar [46] extended previous work to the vibration of concentric annular cylindrically orthotropic plates.

For cylindrically anisotropic plates with small flexural rigidities and subject to high in-plane loads, the numerical-perturbation method of matched asymptotic expansions was used to study the free vibrations [48].

EFFECTS OF THICKNESS-SHEAR DEFORMATION ON SMALL-DEFLECTION MOTION

It has long been accepted that thickness-shear deformation plays a prominent role in sandwich panels; i.e., those with one or more thick, highly flexible cores and one or more thin, relatively rigid facings. However, this effect is often important in composite-material laminated panels as well; i.e.; those with all layers of equal thickness. In fact, it is important even in single-layer panels of composite material due to their very low ratio of shear modulus to Young's modulus as compared to isotropic materials.

Symmetric Laminates

A Galerkin-type approach was used to study the free vibration of rectangular plates of rectangularly orthotropic material and subjected to any combination of simply supported, elastically supported, or clamped edge conditions [49]. Rotatory inertia was included.

Hinton extended the finite-strip method to the symmetrically laminated anisotropic case [50] and also developed an analogous finite element [51].

The Mindlin-Goodman procedure [53] was used to obtain a solution for the transient response of an infinite long plate strip simply supported along the sides [52]. The effects of pulse shape (rectangular, triangular, and sinusoidal) and pulse dwell time on dynamic load factor, maximum deflection, maximum bending stress, and maximum interlaminar shear stress were investigated. A somewhat similar analysis has been reported for the plane-strain case in the presence of residual thermal stresses [54]. The results indicated that thermal stresses are usually detrimental to dynamic behavior.

Unsymmetric Laminates

There has been considerable activity in the case of free vibration of rectangular-planform plates. Bert and Chen [55] presented a closed-form solution for a certain kind of simply supported edges. The results of a mixed finite element to this problem [56] were in good agreement. An eight-node, 40-degree-of-freedom thick-plate element was in good agreement with experimentally determined natural frequencies [57]. The British Program VIPASA has been extended [58].

Venkatesan and Kunukkasseril [59] analyzed the free vibration of a circular-planform plate laminated of different isotropic materials. Symbolic manipulation techniques in conjunction with a Rayleigh-Ritz analysis of a clamped elliptic plate have been used [60, 61].

A thick multilayer laminate theory has been formulated and used to predict the natural frequencies of a simply-supported rectangular plate [62]. Bending, traction, and shear effects are included for each layer, and continuity of stresses and displacements at the interlaminar interfaces is maintained. Dong and Pauley [63] used elements through the thickness to investigate plane wave propagation in thick, laminated plates.

The forced response of a symmetric cross-ply substrate was analyzed with a constrained damping layer [64].

Sandwich Panels

Earlier literature on dynamics of sandwich panels is extensive [1, 65]; recent analyses have thus been directed toward specialized aspects – for example, the effect of thickness-normal flexibility of core material [66]. The effect on impact is somewhat analogous to the impact of a thickness-normal-rigid sandwich plate on a free surface of liquid.

A rectangular sandwich panel with unsymmetric orthotropic facings was subjected to a time-harmonic point loading normal to the plate [67]. Calculations for a sandwich with one facing of aluminum, the other of steel, and a polyvinyl chloride core indicated that the effect of symmetric modes on antisymmetric ones was not pronounced.

Lee and Chang [68] investigated the dispersion relations for a sandwich with symmetric facings and isotropic materials throughout. The material arrangement was typical of electrostatically charged precipitator plates; i.e., a heavy,stiff core (the plate itself) and a light, flexible facing (coatings of dust particles adhering to the plate). This is the converse of the material arrangement typical of structural applications.

Relatively compact electronic devices are sometimes mounted on sandwich panels. It is of interest, therefore, to study the effect of a mass-spring-dashpot system attached at an arbitrary point. The damped free vibration of one such system had a rectangular plate with symmetric orthotropic facings and orthotropic core [69].

Chen and Carne [70] conducted a finite-element and experimental investigation of an open sandwich panel consisting of a single flat plate with a trapezoidal corrugated member attached. An epoxy-adhesive bonded structure had a greater stiffness than a spot-welded one.

A number of recent papers have been concerned with the damping behavior of sandwich panels. In an experimental investigation of a foam-core panel, and on the basis of a beam analysis, the system was believed to behave as a distributed-parameter tuned dynamic absorber [71]. At low frequencies, the behavior was dominated by thickness-normal action analogous to a spring-mass system; at higher frequencies the system behaved more like a homogeneous beam.

Damping has been incorporated into dynamic analyses of sandwich panels in three ways in recent work:

- Use of single composite loss factor [72, 73]. The main disadvantage of this approach is that the composite loss factor is most conveniently determined empirically from experiments.
- Use of a complex-modulus approach for the core-material shear moduli only [74, 75]. The main disadvantage of this approach is that it categorically neglects any damping in the facings.
- Use of a complex-modulus approach for the Young's modulus of the facing material as well as the shear modulus of the core material [76]. In any case, this is the most accurate model, especially with facings of polymer-matrix composite material. Unfortunately, however, the work was limited to isotropic facings and an isotropic core.

Flutter of a sandwich panel was analyzed [77] using the finite-element method.

EFFECTS OF GEOMETRIC NONLINEARITY

Of the two sources of nonlinearity in the dynamics of composite-material plates, the most common is geometric nonlinearity at finite deflections due to the stiffening membrane action when the plate edges are prevented from undergoing any in-plane displacement. The effect is a hardening-spring action that causes the natural frequencies to increase as the amplitude of motion is increased.

Thin Laminates

A variety of solutions for uniform-thickness, rectangularly orthotropic thin plates undergoing large-amplitude vibration have recently appeared. It is gratifying that only one [78] of these investigations used the overworked Berger's hypothesis, which has been shown to be unreliable in many cases [1]. Various methods have been applied to rectangular orthotropic plates: the Galerkin technique [79, 80], the perturbation method [81], and a simplified 16-degree-of-freedom rectangular conforming finite element [82]. The latter kind of element has also been used with cylindrically orthotropic plates [83].

Radially tapered cylindrically orthotropic circular plates were treated analytically [84] and with finite elements [85]. Annular plates with two different nonlinear tapers were used to obtain solutions by a combination of the Kantorovich method and the Newton-Raphson iteration scheme [84]. A linear-tapered solid circular plate was treated with finite elements [85]. Two large-amplitude analyses of unsymmetric laminates have appeared [86, 87]. In one case antisymmetric cross-ply rectangular plates with two opposite edges simply supported and the other two edges clamped were studied using the Galerkin technique. Both stress-free and immovable in-plane edge conditions were included.

Most of the analyses mentioned above involved only one term in position and thus neglected the effects of modal coupling. An exception is the work of Chia and Prabhakara [87], who presented multiple-mode solutions for rectangular plates of both antisymmetric angle-ply and antisymmetric cross-ply with both simply supported and clamped edges. Although the effect of modal coupling on the nonlinear frequencies of isotropic plates is not significant, it can be significant for composite panels, especially clamped-edge high-modulus laminates.

Thick Laminates and Sandwich Panels

Sathyamoorthy [88, 89] included the effects of thickness shear deformation and rotatory inertia for symmetrically laminated rectangularly orthotropic plates. Unfortunately, he used the Berger hypothesis, rather than working directly with the dynamic version of the von Karman equations. The Berger hypothesis was also used to derive a relatively simple frequency expression for a simply supported rectangular plate of isotropic symmetric sandwich construction [90].

In an analysis of the nonlinear dynamic snap-through symmetric buckling of a simply supported rectangular plate of isotropic symmetric sandwich structure, the plate is assumed to have a slight initial curvature [91]. A double Fourier series in position and Houbolt's timewise integration scheme was used.

EFFECTS OF NONLINEAR MATERIAL BEHAVIOR

Physical (material behavior) nonlinearity may be due to nonlinear stress-strain relations such as exemplified by ordinary yielding or that due to the softening nonlinearity encountered in the shear stress vs. shear strain relation for certain composites.

Zak [92] considered physical nonlinearity in the form of orthotropic elastoplastic material behavior. He also considered lamination effects, thickness shear deformation, and geometric nonlinearity. His analysis is based on the use of numerical timewise integration and a uniform-thickness quadrilateral finite element stacked in the thickness direction to represent the composite layers.

Sun and Shafey [93] considered material-behavior nonlinearity in the form of an additional cubic shear-strain term in the thickness shear stress-strain relation. They investigated harmonic wave propagation, formation of shock waves, and free vibrations. A much more elaborate analysis included up to fourth-order elastic constants – i.e., up to cubics of the strains [94].

TRENDS AND SUGGESTIONS FOR FUTURE RESEARCH

The following trends are notable since the first review of this series [1]:

- Use of the finite element method for both linear and nonlinear problems [95]
- Increased study of such practical complications as cutouts, attached mass-spring damper systems, elastically supported edges, and random excitation
- Increased activity in the area of geometrically nonlinear problems

The author believes that the following aspects should be investigated more fully:

- More general lamination schemes with attention toward their use in optimal design syntheses for such multiple conditions as flight loadings, noise, buckling, and flutter
- More realistic material models, with respect to both effects of frequency and temperature on stiffnesses and damping and to fatigue

properties and even the nonlinearity of the stress-strain relations

- Interactions between vibration and material flaws, for example, crack propagation under various kinds of vibratory excitations
- Experimental verification of analyses, as well as vibration testing as a system identification technique

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Composite materials, finite elements, free vibration, forced vibration, laminated plates, natural frequencies, nonlinear vibration, plate structures, sandwich plates, thickness shear deformation.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper is a survey of literature concerning dynamics of plate-type structural elements of either composite material or sandwich construction. Papers which appeared from late 1976 through early 1979 are reviewed. Special attention is given to anisotropic plates, laminated plates, thick and sandwich plates, and nonlinearities. Free vibration and various kinds of forced vibration are treated, as well as classical and finite-element methods of analysis.